* VECTORS

* Scalar Quantities

A Physical quantity which can be described completely by its magnitude only and does not require a direction is called scalar quantity.

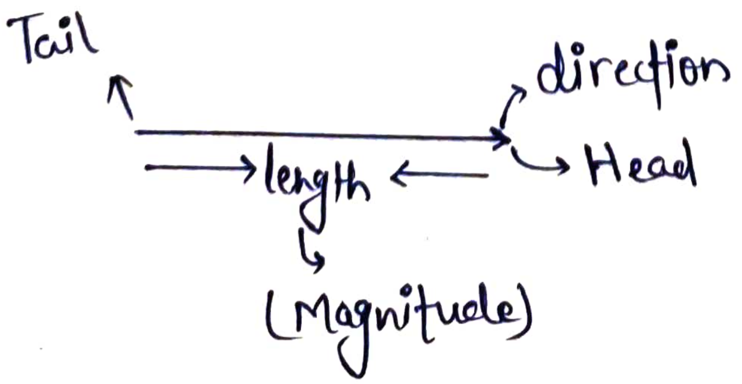
Ex:- Mass, Volume, Density, Time, Temperature, Distance, speed etc.

* Vector quantities

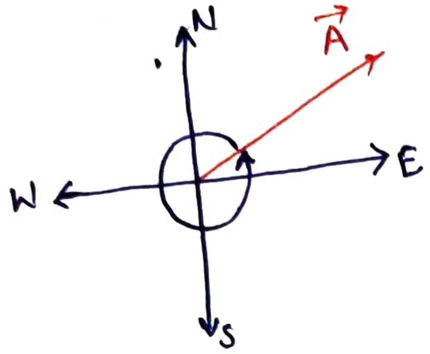
The Physical quantities which have both magnitude and direction and obey the laws of Vector addition are called Vector quantities.

Ex:- Displacement, Velocity, Acceleration, Force, Momentum, etc.

* Representations of Vectors:-



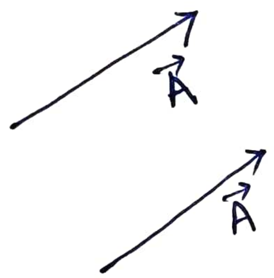
* On rotating 2π, vector does not change.

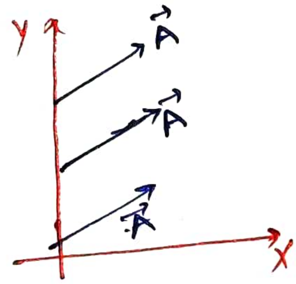


$\pi \rightarrow 180°$

$2\pi \rightarrow 360°$

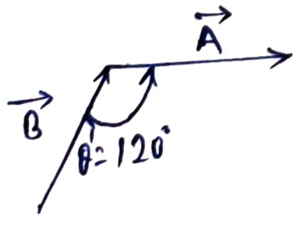
* A Vector can shift parallel to itself



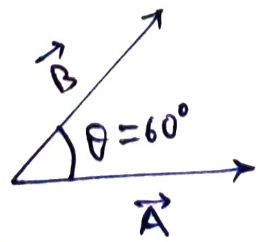


* Angle between two vectors (θ) :-

Angle between two vectors is the smaller of two angles between the vectors when they are placed tail to tail.



$\Rightarrow$



$\boxed{0° \leq \theta \leq 180°}$

* Angle between $\vec{A}$ and $\vec{B}$ is 60° not 120°. Because, their tails are not together while in fig., they are drawn correctly.

* Types of Vectors :-

⊛ Polar vectors :-

These are the vectors which have a initial starting point or a point of application.

Ex:-

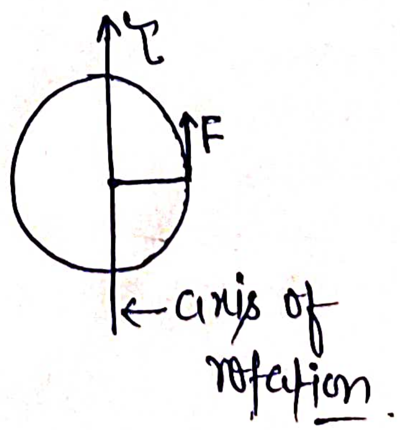
Displacement, Velocity, force, etc.

⊛ Axial Vectors :-

These are the vectors which represent rotational effect and act along the axis of rotation in accordance with right hand screw rule or right hand thumb rule.

Ex:-

Angular velocity (ω), Torque (τ), etc.



* Equal Vector:-

Two Vectors are
(i) equal Magnitude
(ii) Same direction

$\vec{a}$

$\vec{b}$

$\vec{a} = \vec{b}$

* Parallel Vectors:-

Those vectors which are parallel to each other.

* along the same direction

$\vec{a}$

$\vec{b}$

$\theta = 0^\circ$

* Antiparallel Vectors:-

Those vectors which are parallel to each other but opposite in direction.

$\vec{a}$

$\vec{b}$

$\theta = 180^\circ$

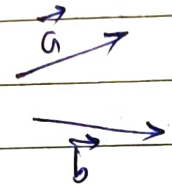
* Collinear Vectors:-

Those vectors which lie on same line.

$\vec{a}$ $\vec{b}$

* Coplanar vectors :-

Those vectors which lie in the same plane.



- Two vectors will always be coplanar
- Three vectors may or may not be coplanar.

* Negative vector :-

- equal magnitude but opposite direction

$\vec{a} = 2m/s$ east

$\vec{b} = 2m/s$ west

$|\vec{a}| = |\vec{b}|$

$\vec{a} = -\vec{b}$

* Null vector :-

Whose magnitude is zero.

$\vec{a} - \vec{a} = \vec{0}$ ↳ zero vector (or) Null vector.

- direction is arbitrary.

* Unit vector:-

A vector whose magnitude is one unit and points in a particular direction, is called unit vector.

* It is represented by $\hat{A}$ (A cap or A hat).

$$\boxed{\vec{A} = \hat{A}|\vec{A}|}$$

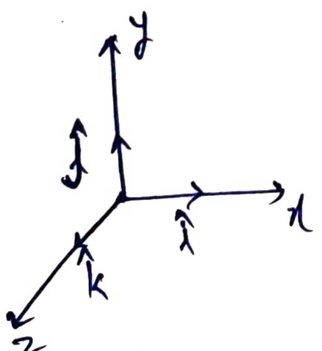
$$\boxed{\hat{A} = \frac{\vec{A}}{|\vec{A}|}}$$

$$\hat{A} = \frac{\text{Vector}}{\text{Magnitude of the Vector}} = \frac{\vec{A}}{|\vec{A}|}$$

Here, $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$\therefore \vec{A} = \hat{A}|\vec{A}|$

$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$



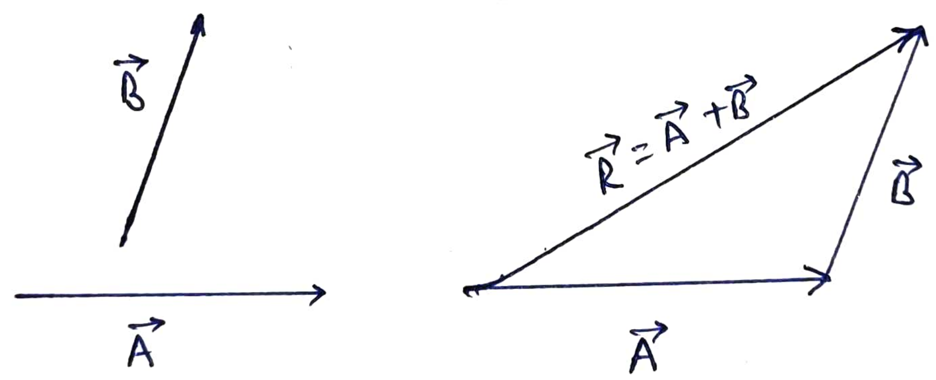
In cartesian coordinates $\hat{i}, \hat{j}, \hat{k}$ are the unit vectors along x-axis, y-axis and z-axis, respectively.

where, $|\hat{i}| = |\hat{j}| = |\hat{k}| = 1$

* Laws of Vector Addition :-

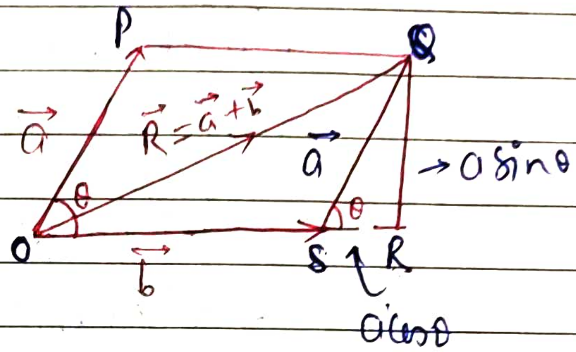
* Triangle law of Vector Addition :-

If two vectors can be represented both in magnitude and direction by the two sides of a triangle taken in the same order, then their resultant is represented completely, both in magnitude and direction, by the third side of the triangle taken in the opposite order.



* Parallelogram Method :-

The two vectors are represented by two adjacent sides of a parallelogram then resultant is given by the diagonal passing through their common point.



From Δ QSR

$$\frac{QR}{QS} = \sin\theta$$

$$\frac{QR}{a} = \sin\theta$$

$$\boxed{QR = a\sin\theta}$$

Again,

from Δ QSR

$$\frac{SR}{QS} = \cos\theta$$

$$\frac{SR}{a} = \cos\theta$$

$$\boxed{SR = a\cos\theta}$$

In Δ OQR

$$OQ^2 = QR^2 + OR^2$$

$$OQ^2 = QR^2 + (OS + SR)^2$$

$$OQ^2 = QR^2 + OS^2 + SR^2 + 2\cdot OS\,SR$$

$$OQ^2 = \underline{QR^2 + SR^2} + OS^2 + 2\cdot OS\cdot SR$$

$$OQ^2 = QS^2 + OS^2 + 2\cdot OS\cdot SR$$

Putting values

$$R^2 = a^2 + b^2 + 2\cdot b\cdot a\cos\theta$$

$$R^2 = a^2 + b^2 + 2ab\cos\theta$$

$$R = \sqrt{a^2 + b^2 + 2ab\cos\theta}$$

When $\vec{a}$ and $\vec{b}$ are in the same direction:

$$\theta = 0^\circ$$

$$R = \sqrt{a^2+b^2+2ab\cos\theta}$$
$$= \sqrt{a^2+b^2+2ab\cos(0)}$$
$$= \sqrt{a^2+b^2+2ab}$$
$$= \sqrt{(a+b)^2}$$
$$\boxed{R = a+b}$$

When $\vec{a}$ and $\vec{b}$ are in opposite direction:

$$\theta = 180^\circ$$

$$R = \sqrt{a^2+b^2+\cancel{\cos}\,2ab\cos(180^\circ)}$$
$$= \sqrt{a^2+b^2+2ab(-1)}$$
$$= \sqrt{a^2+b^2-2ab}$$
$$= \sqrt{(a-b)^2}$$
$$\boxed{R = a-b}$$

If $\vec{a}$ and $\vec{b}$ are at right angle to each other:

$$\theta = 90^\circ$$

$$R = \sqrt{a^2+b^2+2ab\cos(90^\circ)}$$
$$= \sqrt{a^2+b^2+0}$$
$$\boxed{R = \sqrt{a^2+b^2}}$$